GENERAL

linear programming word problems and solutions

Published: September 3, 2026 | Index ID: #-

Linear Programming Word Problems and Solutions: A Complete Guide

Linear programming (LP) is the backbone of modern decision making in business, engineering, logistics, and many other fields. At its core, LP transforms everyday resource allocation and optimization challenges into mathematical models that can be solved efficiently. Whether you are a student tackling exam problems, a professional looking to streamline operations, or simply curious about how math can drive real world decisions, this guide will walk you through the fundamentals of linear programming word problems and provide step by step solutions.

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What Is Linear Programming?

Linear programming is a mathematical technique used to determine the best possible outcome—such as maximum profit or lowest cost—within a set of linear constraints. The “linear” part means that both the objective function (the thing you want to maximize or minimize) and the constraints (the limitations you must respect) are linear equations or inequalities.

Typical LP problems involve:

- Decision Variables: Quantities you can control (e.g., units of product to produce).
- Objective Function: A linear expression that represents the goal (e.g., maximize revenue).
- Constraints: Linear inequalities that limit feasible solutions (e.g., resource limits).

When you see a word problem that asks you to allocate resources, schedule tasks, or distribute goods, chances are you can model it with linear programming.

Key Components of a Word Problem

Before you jump into algebra, carefully dissect the problem statement. Most linear programming word problems share a common structure:

1. Identify the decision variables. Ask yourself: “What is the quantity I can control?” For instance, if a factory produces two products, the variables might be the number of units of each product.
2. Formulate the objective function. Determine what you are trying to maximize or minimize. This could be profit, cost, time, or any other measurable quantity.
3. List the constraints. These are the restrictions imposed by resources, capacities, or other limits. Translate

each textual constraint into a linear inequality or equality.

4. Check the feasibility. Ensure that the constraints are consistent and that a solution exists.

Let’s see how these components play out in a concrete example.

Solving Linear Programming Problems Step by Step

Below is a systematic approach that you can follow for almost any linear programming word problem.

Step 1: Read and Understand the Problem

Take your time to read the problem multiple times. Highlight key numbers and phrases such as “at least,” “no more than,” “each unit requires,” and “total cost.”

Step 2: Define Decision Variables

Assign a variable to each controllable quantity. Use clear, descriptive symbols. For example:

- x = number of units of Product A
- y = number of units of Product B

Always state the domain of each variable (usually non negative integers or real numbers).

Step 3: Construct the Objective Function

Write the linear expression that represents the goal. If the problem asks to maximize profit, add the profit per unit times the number of units:

Maximize Z = 5x + 4y

For a minimization problem, the process is identical; just change the word “Maximize” to “Minimize.”

Step 4: Translate Constraints

Convert every resource limitation or requirement into a linear inequality or equality.

Examples:

- Resource A: 2x + 3y "d 100
- Resource B: 4x + y "d 80
- Non negativity: x "e 0, y "e 0

Step 5: Solve the System

There are several ways to solve the LP:

- Graphical Method: Works well for two variables. Plot constraints, find feasible region, evaluate objective function at corner points.
- Simplex Method: A systematic algorithm for larger problems. Most textbooks and software implement this.
- Software Tools: Excel Solver, MATLAB, Python (PuLP, SciPy), R (lpSolve), Gurobi, CPLEX.

After solving, interpret the solution in the context of the original problem.

Step 6: Verify and Interpret

Check that the solution satisfies all constraints. Compute the objective value. If the problem is a maximization, confirm that no other feasible point yields a higher value.

Common Types of Constraints

In linear programming word problems, constraints typically fall into these categories:

- Resource Constraints: Limited amounts of raw materials, labor, machine hours, etc.
- Demand Constraints: Minimum or maximum production to meet market demand.
- Capacity Constraints: Physical or logistical limits, such as warehouse space or transport capacity.
- Budget Constraints: Financial limits on spending.
- Quality Constraints: Minimum or maximum acceptable levels of quality or performance.
- Regulatory Constraints: Legal or environmental restrictions.

When translating these into inequalities, pay close attention to words like “at least,” “no more than,” or “exactly.” For example, “at least 10 units” becomes x "e 10, while “no more than 20 units” becomes x "d 20.

Real World Examples with Solutions

Example 1: Production Planning for a Bakery

A bakery produces two types of bread: sourdough (S) and rye (R). The bakery wants to maximize daily profit. The following information is available:

- Profit per loaf: $1.20 for sourdough, $1.00 for rye.
- Flour requirement: 0.5 kg per sourdough, 0.4 kg per rye.
- Yeast requirement: 0.1 kg per sourdough, 0.15 kg per rye.
- Daily supply: 60 kg of flour, 15 kg of yeast.
- Demand: At least 30 loaves of sourdough and at most 70 loaves of rye.

Formulate and solve the LP.

Decision variables:

- S = number of sourdough loaves
- R = number of rye loaves

Objective function (maximize profit):

Maximize P = 1.20S + 1.00R

Constraints:

1. Flour: 0.5S + 0.4R "d 60
2. Yeast: 0.1S + 0.15R "d 15
3. Demand sourdough: S "e 30
4. Demand rye: R "d 70
5. Non negativity: S "e 0, R "e 0

Because we have two variables, we can use the graphical method.

1. Plot each inequality on the S–R plane.

2. Identify the feasible region (the intersection of all constraints).

3. Evaluate the objective function at each corner point of the feasible region.

Corner points:

- (30, 0) – meets flour and yeast constraints? 0.5(30)+0.4(0)=15"d60, 0.1(30)+0.15(0)=3"d15. Profit = 1.20(30)+1.00(0)=36.

- (30, 70) – check constraints: 0.5(30)+0.4(70)=15+28=43"d60, 0.1(30)+0.15(70)=3+10.5=13.5"d15. Profit = 1.20(30)+1.00(70)=36+70=106.
- (60, 0) – not feasible because S"e30, but check: 0.5(60)=30"d60, 0.1(60)=6"d15. Profit = 1.20(60)=72.
- (60, 70) – violates flour: 0.5(60)+0.4(70)=30+28=58"d60, yeast 0.1(60)+0.15(70)=6+10.5=16.5>15, so infeasible.

The optimal point is (30, 70) with a profit of $106. The bakery should produce 30 sourdough loaves and 70 rye loaves daily.

Example 2: Transportation Problem

A company must ship products from two factories (F1 and F2) to three warehouses (W1, W2, W3). Shipping costs per unit are given in the table below. Each factory has a supply limit, and each warehouse has a demand requirement.

Factory / Warehouse	W1	W2	W3
F1	$4	$6	$8
F2	$5	$4	$7

Supply limits: F1 = 100 units, F2 = 150 units.

Demand requirements: W1 = 80 units, W2 = 70 units, W3 = 90 units.

Formulate as an LP and find the minimum shipping cost.

Decision variables:

- x • • = units shipped from F1 to W1
- x • , = units shipped from F1 to W2
- x • ƒ = units shipped from F1 to W3
- x , • = units shipped from F2 to W1
- x , , = units shipped from F2 to W2
- x , ƒ = units shipped from F2 to W3

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